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Title: **A Nonlocal Progressive Damage Model for Composite Materials**

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ABSTRACT

Fiber reinforced composite materials are widely used in aerospace structures due to their high specific stiffness and strength. These materials exhibit complex deformation, damage and failure mechanisms under high strain rate loading conditions. Many different failure criteria have been proposed in literature to describe the damage initiation and evolution of fiber reinforced composite materials. Most of these damage models are based on a local framework. In a local framework, the material behavior is based on a point-wise constitutive relation which is independent of the effect of the surrounding points. These models include stiffness degradation when damage initiates in the material, which within a local framework leads to a highly mesh dependent result. This could be overcome using a nonlocal finite element approach. In the current work, a nonlocal formulation based on the work by Andrade et al [1] is adapted to describe composite material behavior. The LS-DYNA rate dependent progressive damage model, MAT162 is employed to detect the initiation and evolution of damage in orthotropic composite materials. This model is developed as a FORTRAN user material subroutine in the LS-DYNA environment. The results of MAT162 with and without the nonlocal formulation are compared at different mesh densities to validate the model. The numerical analysis exhibits the advantages of the nonlocal formulation.

1. INTRODUCTION

To simulate the progressive damage and failure of fiber-reinforced composite materials [1], damage initiation and propagation criteria are required in order to assess, predict and determine the damage in the material and the softening behavior after damage initiates. These damage laws utilize the concepts of stress or strain to evaluate the damage initiation and progression [2]. The MAT162 model available in LS-DYNA is one such model which can be used to simulate the progressive failure of unidirectional and fabric layer composite materials under a wide range of loading rate conditions. This

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material model was developed based on orthotropic material models (MAT 002 and MAT 059) [3]. It is based on the explicit formulation and a local framework and it has been developed for single integration point solid elements. A number of studies have been carried out which verify and employ this material model [4–8].

In a local framework, the material behavior is based on a point-wise constitutive relation which is independent of the effect of the surrounding points. That is, stress at a point depends only on the strain at that point. Damage models such as MAT162 include stiffness degradation when damage initiates in the material. Within a local framework this leads to a highly mesh dependent result because of the inherent localization. This is greatly dependent on the spatial discretization, which can lead to a loss of accuracy due to the fact that as a mesh gets finer, the dissipative variables get unrealistically concentrated into the smaller elements [9].

To overcome this issue, a nonlocal theory has been proposed which incorporates an intrinsic length scale into the original continuum theory [10]. This circumvents the mesh dependency issue using either gradient or integral formulations to function as a localization limiter for plasticity and damage. The first nonlocal models were proposed for elasticity and was later extended to plasticity by Eringen [11]. Pijaudier Cabot and Bazant [12] developed the first nonlocal damage model in continuum damage mechanics (CDM). In their method, they applied a local averaging operator only to variables which could either grow or not change and were related to the inelastic process. The integral nonlocal enhancement converts the constitutive behavior from a local phenomenon to the integral average of all the elements within a preset radius of influence [2]. This radius of influence in itself can be treated as a material property and has been named 'internal length'. This is an important quantity which is related to the size of the damage zone [13].

This algorithm requires simultaneous access to data from all the integration points in the mesh, which is not a common feature in the finite element (FE) packages commonly used in the industry. Thus, in this work, an approximation of the nonlocal theory which can be used with existing local models was implemented based on the approximation of the nonlocal formulation proposed by Tvergaard and Needleman [14] and extended by Andrade et al [10]. This approximate strategy makes use of a nonlocal factor (which is the ratio of the nonlocal and the local variable), calculated from the previous increment values. This is then applied to determine the approximate nonlocal variable for the current increment. This approximation however, requires a small time step to maintain accuracy and prevent instabilities. The MAT162 model is based on the explicit formulation and thus has a small time step based on the size of the smallest element [15].

The unconventional implementation of the nonlocal method is the reason it is not often employed. In the current work, the approximate nonlocal model along with the MAT162 material model were coupled and developed as a user material subroutine in the FE code LS-DYNA. A detailed description of the implementation of the nonlocal model is provided. Analysis of two cases were carried out to verify the nonlocal model. In the first case a single layer 45 degree lamina and in the second a single layer 0 degree notched lamina was loaded under uniaxial tension. The nonlocal capability was verified by comparing the results with and without the nonlocal approach for both the test cases.

2.THEORY

The MAT162 model has been developed for unidirectional and fabric lamina and is briefly described here. For a detailed description the reader is referred to [16,17]. The damage criteria are based on the ply level stresses ($\sigma_a, \sigma_b, \sigma_c, \tau_{ab}, \tau_{bc}, \tau_{ca}$) and elastic moduli ($E_a, E_b, E_c, G_{ab}, G_{bc}, G_{ca}$). Damage initiates when the failure condition, f_i reaches a value of 1. The damage thresholds, r_i determine damage evolution and are equal to 1 in an undamaged state. The damage initiation and evolution for the unidirectional lamina model is briefly described below:

2.1 Damage initiation

The fiber failure is differentiated into three modes due to tension/shear, compression and crush under pressure.
Fiber Tensile/Shear mode:

$$f_1 - r_1^2 = \left(\frac{\langle\sigma_a\rangle}{S_{aT}}\right)^2 + \left(\frac{\tau_{ab}^2 + \tau_{ca}^2}{S_{FS}^2}\right) - r_1^2 = 0 \tag{1}$$

Fiber Compression mode:

$$f_2 - r_2^2 = \left(\frac{\sigma_a'}{S_{aC}}\right)^2 - r_2^2 = 0 \tag{2}$$

Where,

$$\sigma_a' = -\sigma_a + \langle-\frac{\sigma_b + \sigma_c}{2}\rangle \tag{3}$$

Fiber Crush mode:

$$f_3 - r_3^2 = \left(\frac{\langle p\rangle}{S_{FC}}\right) - r_3^2 = 0 \tag{4}$$

Where,

$$\langle p\rangle = -\frac{\sigma_a + \sigma_b + \sigma_c}{3}$$

Here, the $<>$ denote Macauley brackets, S_{aT} and S_{aC} are the tensile and compressive strengths in the fiber direction, S_{FC} and S_{FS} are the fiber crush and fiber shear strengths respectively.

The matrix failure is assumed to occur on plane parallel to the fibers. The matrix failure is differentiated into two modes, in the perpendicular and parallel planes.
Matrix Transverse Compressive mode:

$$f_4 - r_4^2 = \left(\frac{\langle\sigma_b\rangle}{S_{bC}}\right)^2 - r_4^2 = 0 \tag{5}$$

Matrix Perpendicular mode:

$$f_5 - r_5^2 = \left(\frac{\langle \sigma_b \rangle}{S_{bT}}\right)^2 + \left(\frac{\tau_{bc}}{S'_{bc}}\right)^2 + \left(\frac{\tau_{ab}}{S_{ab}}\right)^2 - r_5^2 = 0 \tag{6}$$

Matrix Parallel mode (Delamination):

$$f_6 - r_6^2 = \left(\frac{\langle \sigma_c \rangle}{S_{cT}}\right)^2 + \left(\frac{\tau_{bc}}{S''_{bc}}\right)^2 + \left(\frac{\tau_{ca}}{S_{ca}}\right)^2 - r_6^2 = 0 \tag{7}$$

where,

$$S_{ab} = S_{ab}^{(0)} + \tan(\varphi)\langle -\sigma_b \rangle$$

$$S'_{bc} = S_{bc}^{(0)} + \tan(\varphi)\langle -\sigma_b \rangle$$

$$S_{ca} = S_{ca}^{(0)} + \tan(\varphi)\langle -\sigma_c \rangle$$

$$S''_{bc} = S_{bc}^{(0)} + \tan(\varphi)\langle -\sigma_c \rangle$$

S_{bT} and S_{cT} are the tensile strengths perpendicular to the fiber direction. S_{ab}, S_{bc}, and S_{ca} are the shear strengths and φ is a material constant.

The effect of strain rate on the properties of the strengths and the moduli is also taken into account. This is done by utilizing a logarithmic strain rate dependent function as shown below.

$$\frac{X_{RT}}{X_0} = 1 + C_{rate} \log \frac{\dot{\bar{\varepsilon}}}{\bar{\varepsilon}_0} \tag{8}$$

Where X_{RT} is the rate dependent property at the strain $\bar{\varepsilon}$, X_0 is the initial property, $\bar{\varepsilon}_0$ is the reference strain rate and C_{rate} is the coefficient for strain rate dependent property.

2.2 Damage evolution

The damage in the material is represented as smeared cracks, wherein only the effect of cracks on the material properties are considered [18]. Thus, the elastic moduli are degraded when damage is found to have initiated, leading to a redistribution of stress in the lamina. The damage evolution is based on the method developed by Matzenmiller et al [19]. A set of damage variables are used (one for each mode of damage) in conjunction with a damage coupling matrix to describe the degradation in the material after damage initiates. These damage variables are then used in the constitutive matrix.

$$\omega_i = 1 - e^{\left(\frac{1}{m_i}\left(1 - r_i^{m_i}\right)\right)} \tag{9}$$

where ω_i represents the damage parameter, r_i is the damage threshold for the corresponding damage mode and m_i are the coefficient for strain softening which determine the shape of the softening after damage initiates.

2.3 Constitutive relation

A linear elastic, transversely orthotropic stress-strain constitutive response is used in the model. The constitutive model is used to calculate the stress in the material using the material properties and the current strain in the material.

$$\{\epsilon\} = [S]\{\sigma\} \ or \ \{\sigma\} = [C]\{\epsilon\} \tag{10}$$

$$[S] = \begin{bmatrix} \dfrac{1}{(1-\omega_1)E_a} & -\dfrac{v_{ba}}{E_b} & -\dfrac{v_{ca}}{E_c} & 0 & 0 & 0 \\ -\dfrac{v_{ab}}{E_a} & \dfrac{1}{(1-\omega_2)E_b} & -\dfrac{v_{cb}}{E_c} & 0 & 0 & 0 \\ -\dfrac{v_{ac}}{E_a} & -\dfrac{v_{bc}}{E_b} & \dfrac{1}{(1-\omega_3)E_c} & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{1}{(1-\omega_4)G_{ab}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \dfrac{1}{(1-\omega_5)G_{bc}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \dfrac{1}{(1-\omega_6)G_{ca}} \end{bmatrix}$$

Where v_{ij} are the Poisson ratio, E$_i$ are the Young's moduli and G$_{ij}$ are the shear moduli.

2.4 Nonlocal formulation

There are several choices for the variables which are to be regularized by the nonlocal procedure. These include the strain tensor, damage variables, plastic strain and other internal variables [13]. The nonlocal variable will also depend on the material and the failure criteria being used. However, the method for the nonlocal regularization is the same for all variables.

For a general case, the nonlocal average at a generic point x, averaged over a fixed volume, V is described as:

$$\overline{z_n}(x) = \int_V \beta(x,\xi)z_n(\xi)dV(\xi) \tag{12}$$

where $\overline{z_n}$ is the nonlocal variable and z_n is the local variable, $\beta(x,\xi)$ is the weighted averaging operator, x is the global coordinate for the local integration point, while ξ is the global coordinate of the integration points in the neighborhood of the local coordinate, within the nonlocal range

$$\beta(x,\xi) = \frac{\alpha(x,\xi)dV(\xi)}{\Omega_r(x)} \tag{13}$$

Here, $\alpha(x,\xi)$ is the weighting function and Ω_r is referred to as the representative volume and is given by

$$\Omega_r(x) = \int_V \alpha(x,\xi)dV(\xi) \tag{14}$$

$$\alpha(x,\xi) = \langle 1 - \frac{||x-\xi||^2}{l_r^2} \rangle^2 \tag{15}$$

The weighting function takes the form of a bell-shaped curve as shown in figure 1, wherein the maximum is at the origin ($x = \xi$) and the function decreases as the distance increases. The Macauley brackets ensure only the integration points which lie in the nonlocal range are selected for nonlocal averaging. The intrinsic/characteristic length, l_r of the material is determined from analyzing the experimental data. This characteristic length determines the nonlocal range for averaging. Ω_r is the region of interest for nonlocal averaging with a radius of l_r.

The approximate formulation based on the work by Tvergaard and Needleman [14] and Andrade et al [10] is employed. A nonlocal factor, K_{nl} is utilized to enhance the local variable in the current time step. The K_{nl} factor is determined as the ratio of the nonlocal variable to the local variable, obtained from the previous increment.
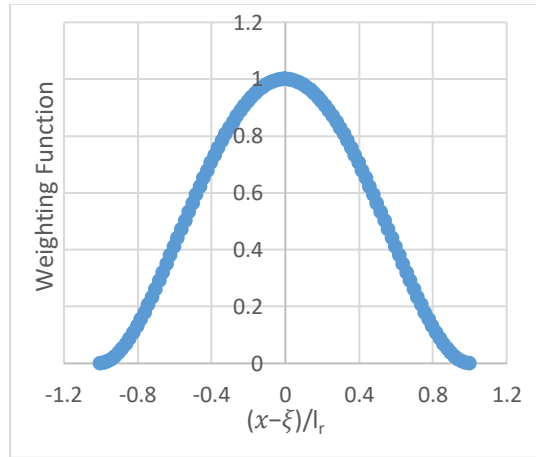
$$K_{nl} = \frac{\overline{z_n}}{z_n} \tag{16}$$

Figure 1. Distribution of the weighting function.

This factor multiplied by the local variable from the current step gives an approximation of the nonlocal variable in the current time step.

$$\overline{z_{n+1}} = K_{nl} * z_{n+1} \tag{17}$$

This method has been interpreted as an "explicit nonlocal integration" [20]. It requires a small time step to insure a stable solution which is a requisite in explicit formulations.

2.5 LS-DYNA implementation

The implementation of the nonlocal model is based on the model implemented in LS DYNA as MAT_NONLOCAL [21] and the models by Andrade et al [2].

$$\overline{z_i} = \int_V \beta(x, \xi) z_n(\xi) dV(\xi) = \frac{1}{\Omega_r} \sum_{j=1}^{nip} z_j \alpha_{ij} V_j \tag{18}$$

where,

$$\Omega_r = \sum_{j=1}^{ngp} \alpha_{ij} V_j$$

$\alpha_{ij} = \alpha(x, \xi)$ is the weighting function relating the integration points of element i and j located at global coordinates $x \ and \ \xi$, z_j is the local variable at j, V_j is the volume of element j and nip is the number of integration points in the nonlocal range of point at i. $\overline{z_i}$ is the nonlocal variable at i.

A vectorized user defined material (UMAT) subroutine was developed for implementing the nonlocal MAT162 model in the LS DYNA environment. Figure 2 shows the algorithm for the subroutine. The nonlocal section is written into the urmathn subroutine.

First, the element nodal connectivity and the nodal coordinate data is obtained. These values are available in the urmathn subroutine as ix1 to ix8 and dm_x respectively. Once the nodal connectivity of an element and the coordinates of the nodes are known, the integration point location can be calculated depending on the type of element formulation. For LS-DYNA solid element formulation 1, constant stress element, there is one integration point per element located at the center of the element. Thus, averaging the nodal coordinates gives the integration point location. The distance between the integration points is computed using the distance between two points formula. If this distance is less than the intrinsic length then the weighting factor is calculated from equation (15) where $x - \xi$ is the distance. These values and the integration point number are saved in an array for each integration point.

The nonlocal variable is then calculated using equation (18). An array is used to save the points in the nonlocal range and their weighting function value. The local variable is saved in the previous increment and the volume of the elements is available in the urmathn subroutine as voln(i) and are saved for every increment. In the current

model, the strains are chosen as the variable to be regularized as the effect of strain rate and all the damage calculations either directly depend on the strain or its derivatives.

The nonlocal factor is calculated as the ratio of the nonlocal variable to the local variable. For the case of nonlocal strain regularization utilizing solid elements, six nonlocal variables and factors are obtained for the six corresponding local strains. These nonlocal factors are then passed to umat.

In the umat, the nonlocal strains are calculated using equation (17). The nonlocal stresses are then calculated using the nonlocal strains and these stresses are used in the MAT162 procedure to determine damage initiation and evolution. After the damage calculations, the local stresses are updated using the local strains and the damage variables calculated from the MAT162 procedure so that LS Dyna can calculate the next increment. A fortran code excerpt of the general nonlocal implementation is provided in Appendix A.
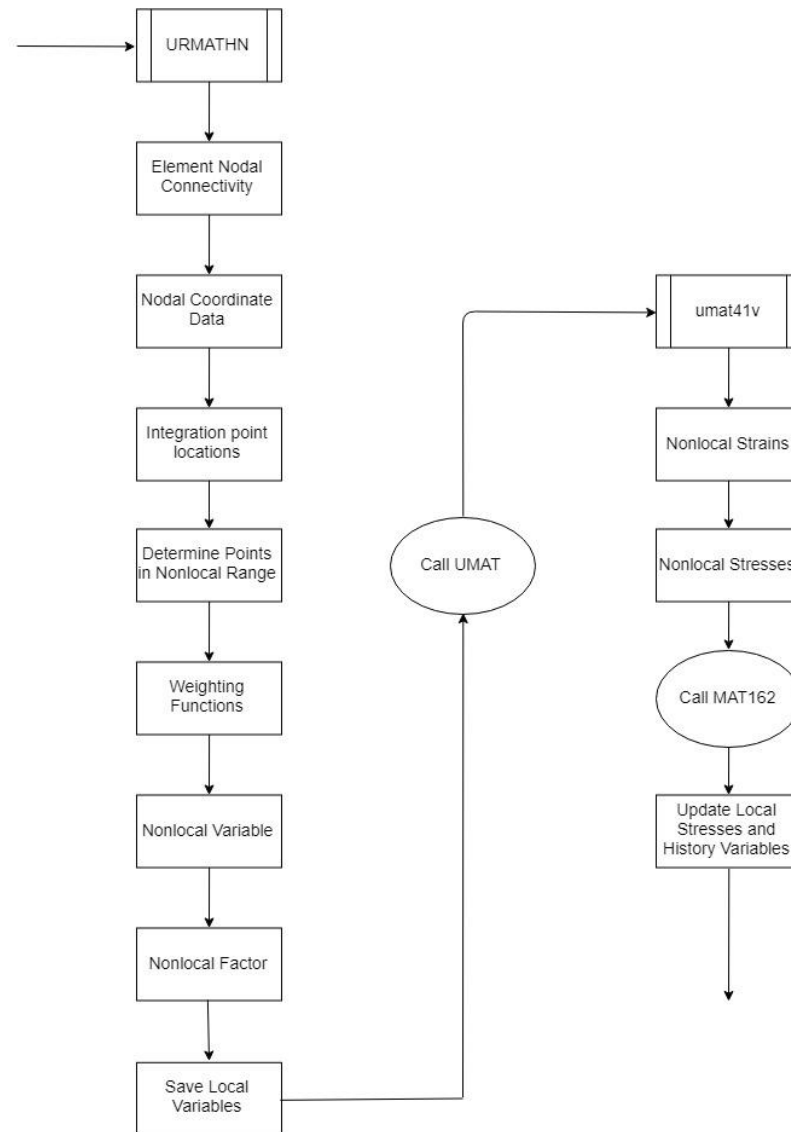
Figure 2. Algorithm for LS-DYNA subroutine.

3. ANALYSIS

Two simple test cases are simulated to demonstrate the nonlocal approach. The models are chosen to show softening due to damage which is a highly mesh dependent parameter. A single layer 45 degree lamina and a single layer 0 degree lamina with a notch are loaded in uniaxial tension. Three different mesh sizes are simulated for both the models. The nonlocal approach requires one addition material property, the intrinsic length (l_r) to be passed to the subroutine. The mesh should be selected such that at least a few elements lie in the nonlocal range. This value is usually based on experimentally observations but, in this work it is arbitrarily selected to demonstrate the nonlocal approach. IM7/977-3 carbon/epoxy composite properties are used for both the test cases and some of the properties are listed in Table I.

Table I. Properties of IM7/977-3 composite in GPa

E_a	E_b	E_c	G_{ab}	G_{bc}	G_{ca}	v_{ab}
273.0	16.0	16.0	13.0	6.25	13.0	0.31

v_{bc}	v_{ca}	X_T	X_C	Y_T	Y_c	S_{ab}
0.31	0.28	2.51	1.68	0.065	0.237	0.113

3.1 Test case 1

In the first test case, a single layer 0-degree lamina with a center notch is loaded in uniaxial tension. Assuming symmetry, only a quarter of the specimen is modeled. The geometry and boundary conditions are shown in figure 3.

A prescribed motion displacement boundary condition is defined at the right end. Symmetry boundary conditions are employed at the bottom and left end. Three meshes of the same model are tested with increasing mesh densities. The meshes have 1298 (Mesh 1), 3985 (Mesh 2) and 19965 (Mesh 3) elements respectively. The size of the biggest element in Mesh 1- the coarsest mesh, is 1 mm. Therefore, the intrinsic length l_r is selected as 1.5mm and is kept the same for all the meshes
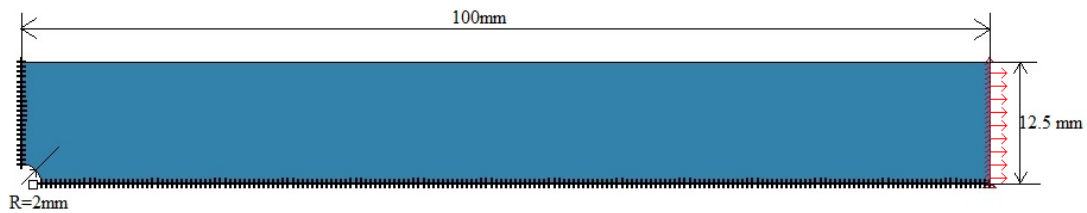

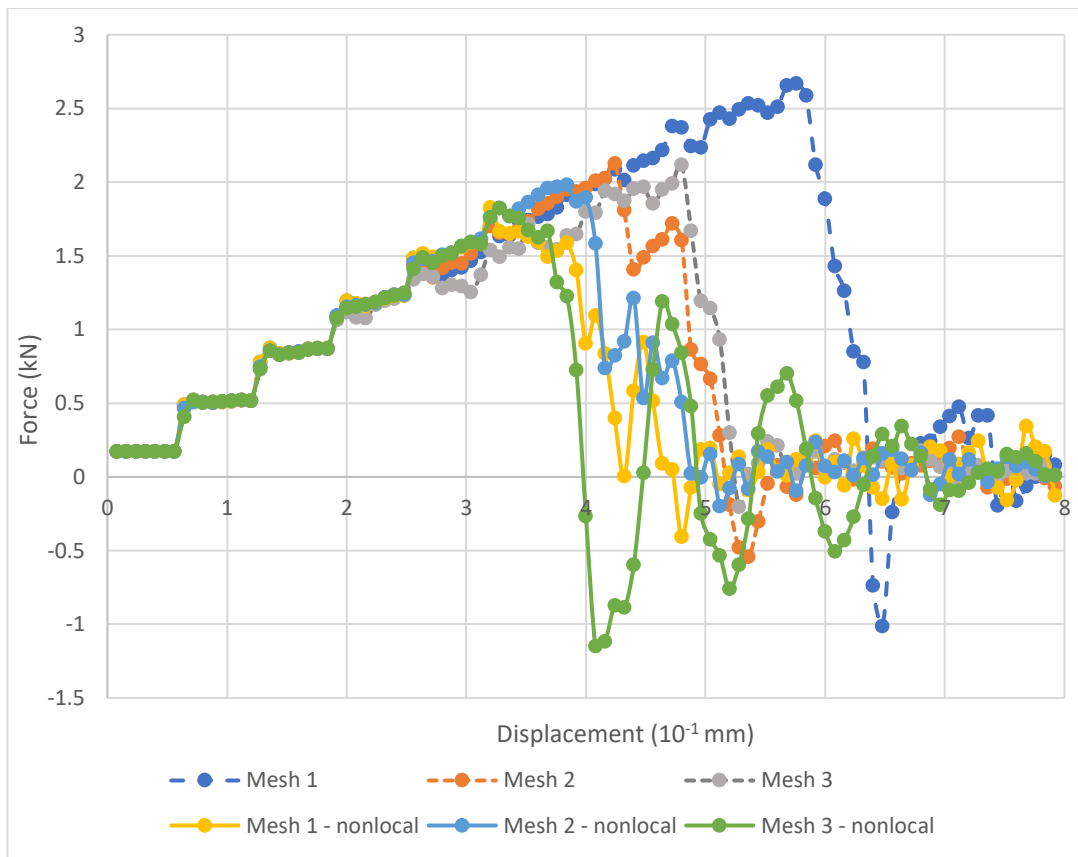Figure 3. Geometry and boundary conditions for case 1.

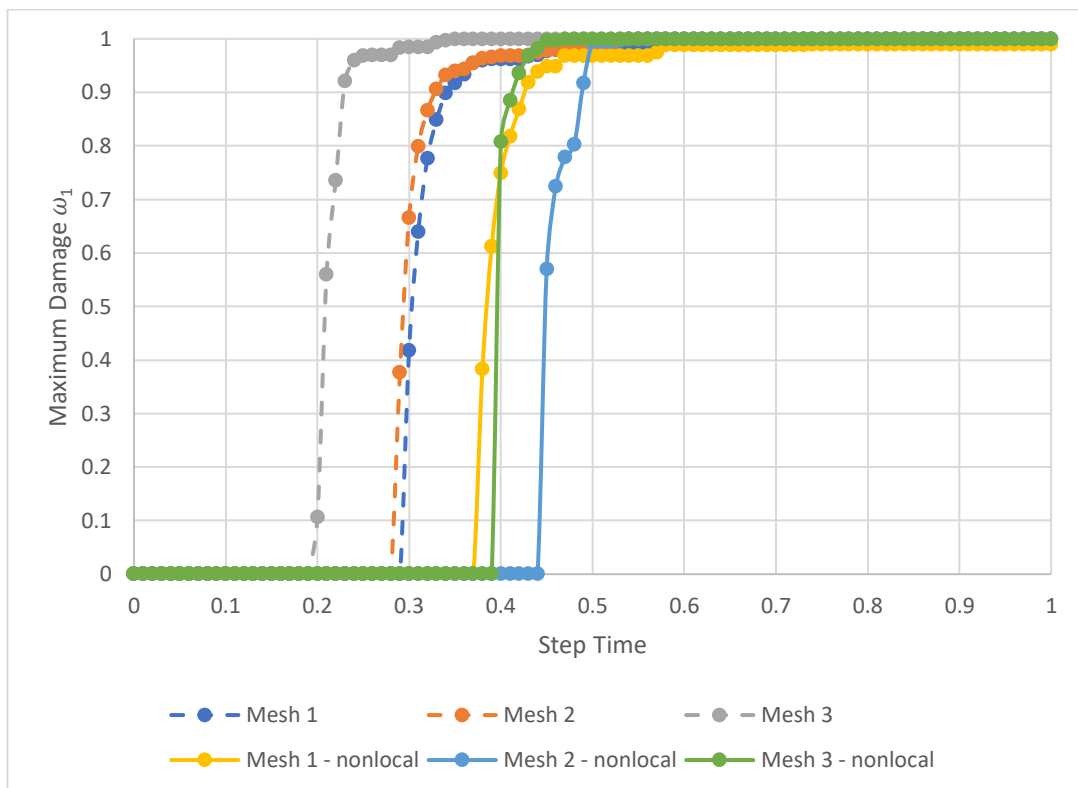Figure 4. Force displacement curve for case 1.

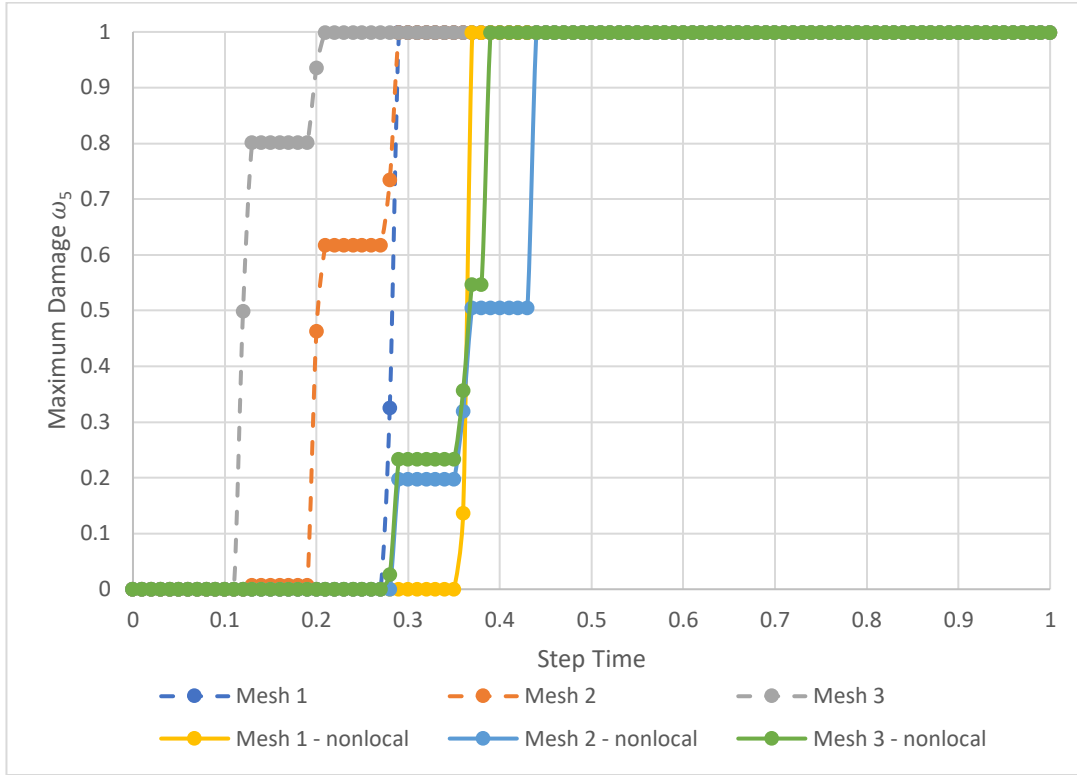Figure 5. Maximum damage due to fiber tension/shear mode.

Figure 6. Maximum damage due to perpendicular matrix mode.

This test case is dominated by matrix perpendicular damage and fiber tension/shear damage. Figure 4 show the force displacement curves for the three meshes with and without the nonlocal method. The presence of the notch leads to stress concentrations in the elements around it, leading to highly mesh dependent results. There is a large variation in the results between the different meshes in the local formulation. The nonlocal results on the other hand are fairly clumped together.

The maximum damage in all the elements of the mesh due to the two damage modes stated above are represented in figures 5 and 6. The damage initiation for the meshes can be observed from these figures. The time of damage initiation decreases as the mesh refinement increases in the local formulation whereas the damage initiates at approximately the same time in the nonlocal model.

3.2 Test case 2

In the second test case, a single layer 45-degree lamina is loaded in uniaxial tension. The geometry and boundary conditions are as shown in figure 7. A prescribed motion displacement boundary condition is defined at the left and right ends. Three meshes of the same model are tested with increasing mesh densities. The meshes have 896 (Mesh 1), 3584 (Mesh 2) and 14336 (Mesh 3) elements respectively. The size of the biggest element in the coarsest mesh, is 2.46 mm. Therefore, the intrinsic length l_r is selected as 3.5 mm and is kept the same for all the meshes

For the 45-degree layer, damage is dominated by matrix failure due to matrix perpendicular mode. Figure 8 shows the force-displacement curve predicted by the models for this test case. Local mesh 3 deviates significantly from the other local meshes whereas the nonlocal meshes are nearly identical for all the meshes.

The variation in the local mesh can be explained by observing the damage growth shown in Figure 9. In all the meshes, local and nonlocal, damage initiates at around 0.5 time units. However, the damage growth is greater in mesh 3. This is due to the fact that as element sizes are reduced, stresses get more concentrated in the elements and failure occurs faster. The damage growth for the nonlocal model is indistinguishable across the meshes.
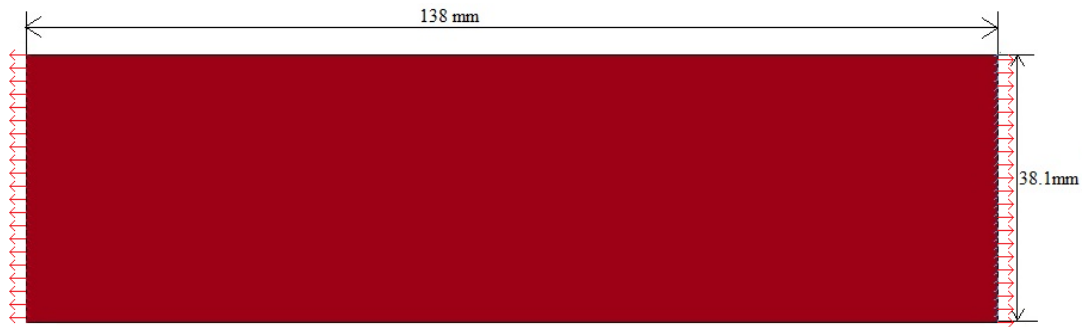

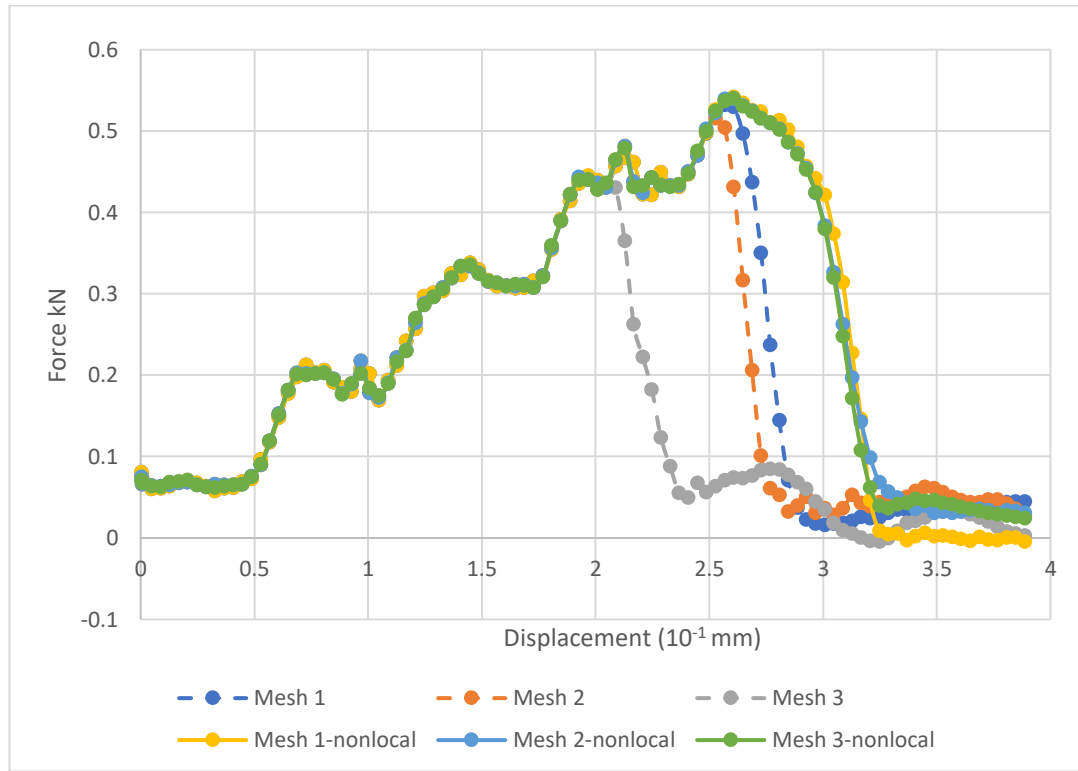Figure 7. Geometry and boundary conditions for case 2.

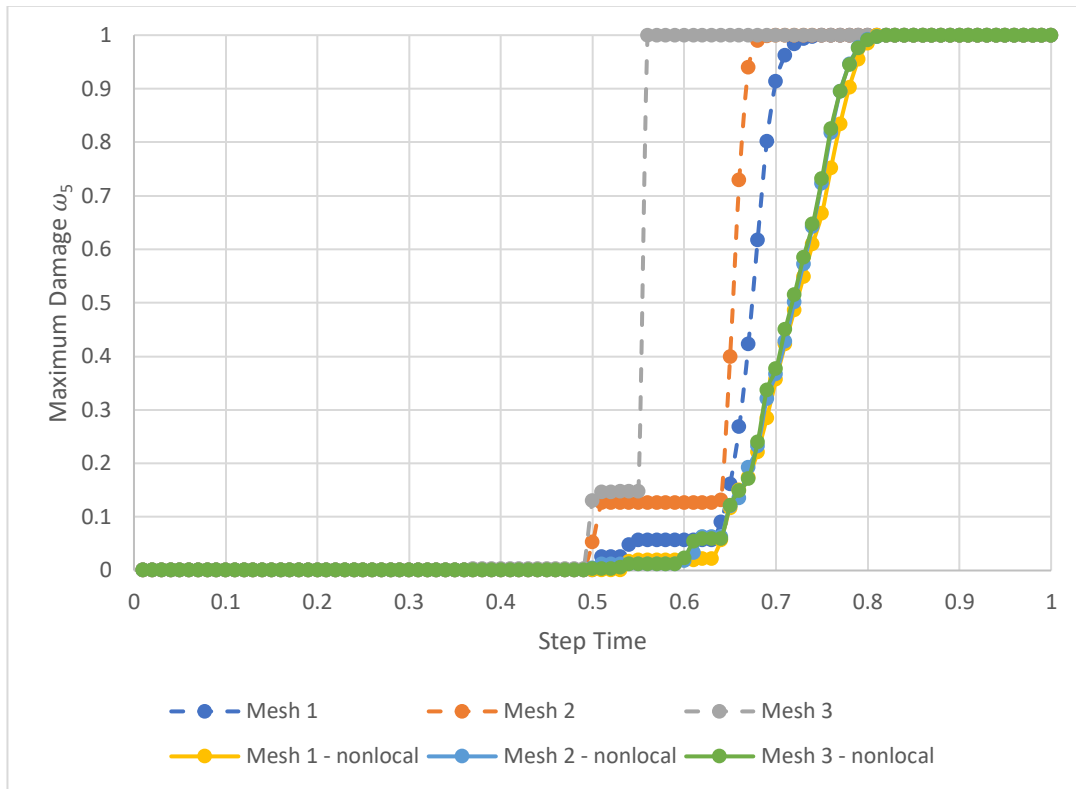Figure 8. Force displacement curve for case 2.

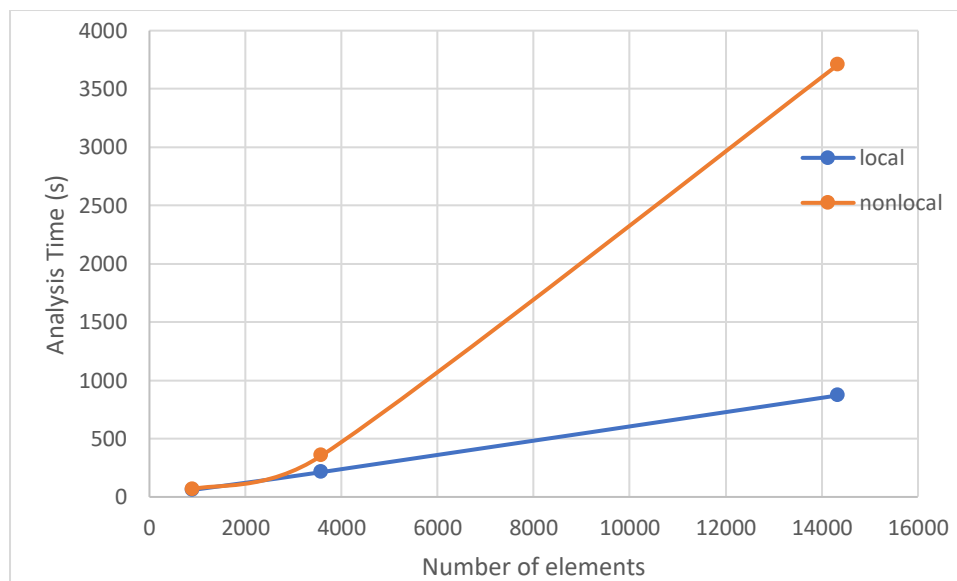Figure 9. Maximum damage due to matrix damage.

Figure 10. Analysis time taken for each mesh.

The total computational analysis time for the nonlocal and the local models of case 2 are compared in figure 10. As the number of elements increases the amount of time

required for the nonlocal analysis increases much more than the local model. For the final mesh the time taken by the nonlocal analysis is four times that of the local analysis. However, as the results are consistent for all the meshes, using a reasonably coarser mesh with the nonlocal approach can assure mesh independent results at low computational cost.

4. CONCLUSION

In this work, a detailed approach to implement the nonlocal progressive damage model for fiber-reinforced composite materials has been presented. The nonlocal approach can be used to regularize any variable. A general outline of the MAT162 model has also been provided. The MAT162 model has been coupled with the nonlocal model and developed as a fortran user material subroutine in the LS-DYNA environment. The model is validated by comparing the results of MAT162 with and without the nonlocal formulation at different mesh densities for two test cases. The numerical analysis exhibits the regularization achievable using a nonlocal approach. It can be concluded that using the nonlocal approach, an analysis can be conducted with reasonably coarser meshes and predict mesh independent – regularized results. The downside to using the nonlocal approach would be the increased analysis time to complete the nonlocal calculations albeit coarser nonlocal meshes can be employed. Additional cases have to be studied to verify the regularization ability across different loading conditions and stacking sequences. The value and effect of the intrinsic length also need to be studied in greater detail.

ACKNOWLEDGEMENTS

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